OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4728

Mechanics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4728
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Monday 20 June 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

1 Two perpendicular forces have magnitudes 8 N and 15 N. Calculate the magnitude of the resultant force, and the angle which the resultant makes with the larger force. **[4]**

2 Particles P and Q, of masses 0.45 kg and m kg respectively, are attached to the ends of a light inextensible string which passes over a small smooth pulley. The particles are released from rest with the string taut and both particles 0.36 m above a horizontal surface. Q descends with acceleration $0.98\,\text{m s}^{-2}$. When Q strikes the surface, it remains at rest.

(i) Calculate the tension in the string while both particles are in motion. **[2]**

(ii) Find the value of m. **[3]**

(iii) Calculate the speed at which Q strikes the surface. **[2]**

(iv) Calculate the greatest height of P above the surface. (You may assume that P does not reach the pulley.) **[3]**

3 A block B of mass 0.8 kg is pulled across a horizontal surface by a force of 6 N inclined at an angle of 60° to the upward vertical. The coefficient of friction between the block and the surface is 0.2. Calculate

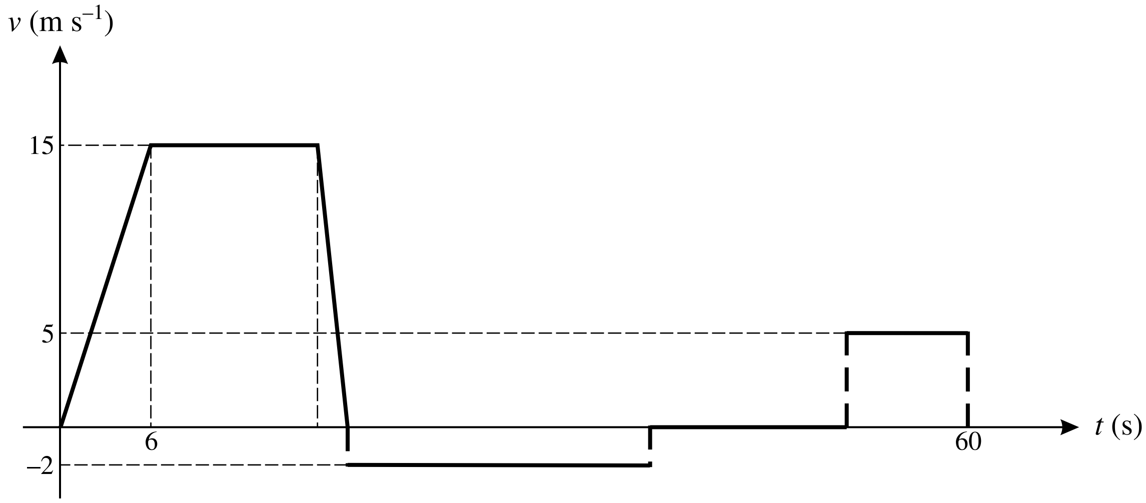
(i) the vertical component of the force exerted on B by the surface, **[2]**

(ii) the acceleration of B. **[4]**

The 6 N force is removed when B has speed $4.9\,\text{m s}^{-1}$.

(iii) Calculate the time taken for B to decelerate from a speed of $4.9\,\text{m s}^{-1}$ to rest. **[4]**

4



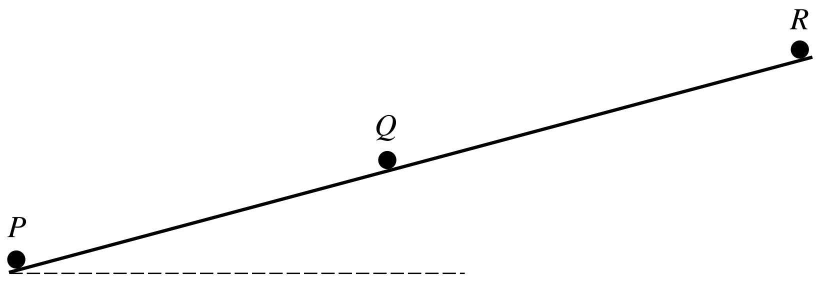
A car travelling on a straight road accelerates from rest to a speed of $15\,\text{m s}^{-1}$ in 6 s. It continues at constant speed for 11 s and then decelerates to rest in 2 s. The driver gets out of the car and walks at a speed of $2\,\text{m s}^{-1}$ for 20 s back to a shop which he enters. Some time later he leaves the shop and jogs to the car at a speed of $5\,\text{m s}^{-1}$. He arrives at the vehicle 60 s after it began to accelerate from rest. The diagram, which has six straight line segments, shows the (t, v) graph for the motion of the driver.

(i) Calculate the initial acceleration and final deceleration of the car. **[3]**

(ii) Calculate the distance the car travels. **[3]**

(iii) Calculate the length of time the driver is in the shop. **[4]**

5

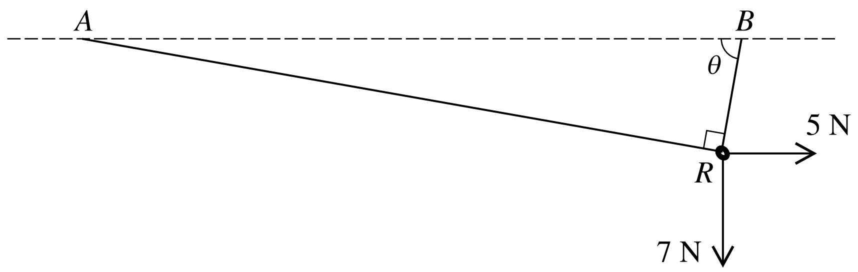


Three particles P, Q and R lie on a line of greatest slope of a smooth inclined plane. P has mass 0.5 kg and initially is at the foot of the plane. R has mass 0.3 kg and initially is at the top of the plane. Q has mass 0.2 kg and is between P and R (see diagram). P is projected up the line of greatest slope with speed $3\,\text{m}\,\text{s}^{-1}$ at the instant when Q and R are released from rest. Each particle has an acceleration of $2.5\,\text{m}\,\text{s}^{-2}$ down the plane.

(i) P and Q collide 0.4 s after being set in motion. Immediately after the collision Q moves up the plane with speed $3.2\,\text{m}\,\text{s}^{-1}$. Find the speed and direction of motion of P immediately after the collision. **[5]**

(ii) 0.6 s after its collision with P, Q collides with R and the two particles coalesce. Find the speed and direction of motion of the combined particle immediately after the collision **[5]**

6



A small smooth ring R of weight 7 N is threaded on a light inextensible string. The ends of the string are attached to fixed points A and B at the same horizontal level. A horizontal force of magnitude 5 N is applied to R. The string is taut. In the equilibrium position the angle ARB is a right angle, and the portion of the string attached to B makes an angle θ with the horizontal (see diagram).

(i) Explain why the tension T N is the same in each part of the string. **[1]**

(ii) By resolving horizontally and vertically for the forces acting on R, form two simultaneous equations in $T\cos\theta$ and $T\sin\theta$. **[4]**

(iii) Hence find T and θ. **[6]**

[Question 7 is printed overleaf.]

7 A particle P is projected from a fixed point O on a straight line. The displacement x m of P from O at time t s after projection is given by $x = 0.1t^3 - 0.3t^2 + 0.2t$.

(i) Express the velocity and acceleration of P in terms of t. **[4]**

(ii) Show that when the acceleration of P is zero, P is at O. **[3]**

(iii) Find the values of t when P is stationary. **[3]**

At the instant when P first leaves O, a particle Q is projected from O. Q moves on the same straight line as P and at time t s after projection the velocity of Q is given by $(0.2t^2 - 0.4)\ \mathrm{m\,s^{-1}}$. P and Q collide first when $t = T$.

(iv) Show that T satisfies the equation $t^2 - 9t + 18 = 0$, and hence find T. **[7]**

Question			Expected Answer	Mark	Rationale/Additional Guidance
1			$R^2 = 8^2 + 15^2$ R = 17 N cosθ = 15/17 θ = 28.1°	M1 A1 M1 A1 [4]	Uses Pythagoras 3 squared terms, addition Uses trig appropriately and targets either angle Accept 28°, 0.49 rad
2	**i**	**Also if in ii**	T – 0.45g = 0.45x0.98 T = 4.85(1) N	M1 A1 [2]	N2L on 0.45 kg, weight - tension and +/-0.98m Not 4.9, 4.8 (4.851 is exact, but 4.85 acceptable) {g=9.81→ T=4.85 or 4.86 or better}
	ii	**Also If in i**	mg – 4.85(1) = 0.98m m = 4.85(1)/(9.8-0.98) or m(g – 0.98) = 4.85(1) m = 0.55 *OR* 0.98 = g (m-0.45)/(m+0.45) m = (g+0.98)/(g-0.98) x 0.45 m = 0.55	M1 A1ft A1 [3] M1 A1 A1	N2L on Q, weight – tension, tension=T(i), and 0.98m Simplified to a single term in m, ft cv(T(i)) art 0.550 {g=9.81→ m=0.55(0) or better} a = g x Δ(masses)/Σ(masses)
	iii		v^2 = (0 +) 2x0.98x0.36 v = 0.84 ms^{-1}	M1 A1 [2]	Uses $v^2 = u^2 + 2as$, a not 9.8, 2as>0, u = 0 or omitted
	iv		$0 = 0.84^2 – 2x9.8s$ (s = 0.036) S = 0.036 + 2x0.36 = 0.756 m	M1 A1 A1 [3]	$0 =(cv(iii))^2 – 2gs$, or t=cv(iii)/g <u>and</u> $s = ut+/-gt^2/2$ May be implied by final answer (eg 0.396) Must be 3 sf (exact) {g=9.81→ s=0.756 or better}

			Frequent mis-read "horizontal/vertical" MR version in {}		Allow all A1 marks in (i) and (ii) *except final A1 in (ii).*
3	**i**		R = 0.8g – 6cos60 {R = 0.8g – 6sin60} R = 4.84 {R = 2.64}	M1 A1 [2]	Resolves vertically, (R=) difference of 2 forces inc. component of 6 Accept 4.8 {2.6} {g=9.81→ R=4.848 {2.65}; accept 4.8 {2.6 or 2.7} }
	ii		Fr = 0.2x4.84 (=0.968) { Fr = 0.2x2.64..(=0.5287..)} 6sin60 - 0.968 = 0.8a {6cos60 – 0.5287.. = 0.8a} a = 5.29 ms^{-2} {a = 3.09 ms^{-2} A0}	M1 M1 A1 A1 [4]	Uses F=0.2(cv(i)) or F=0.2x(R found in (ii) by a method which would be given M1 in (i)) Uses N2L, 3 terms inc. component of 6 Fr need not be evaluated Accept 5.3 {g=9.81→ a=5.28 {3.09 A0} Accept 5.3 {3.1 A0}
	iii		Fr = 0.2x0.8x9.8 (= 1.568) 0.8a = -0.2x0.8x9.8 0 = 4.9 – 1.96t t = 2.5 s	B1 M1* D*M1 A1 [4]	Uses Fr = 0.2x0.8g N2L, Fr only, accept use of Fr from (ii) Accept 0.8a = 0.2x0.8x9.8, (a = (-)1.96) Accept 4.9/1.96, not 0 = 4.9 + 1.96t Accept art 2.50 {g=9.81→ t=2.50 Accept art 2.50}
4	**i**		a = 15/6 or d = 15/2 a = 2.5 ms^{-2} d = 7.5 ms^{-2}	M1 A1 A1 [3]	Uses a = speed change/time Accept -7.5
	ii		T = 6+11+2 (=19) x = 15(11+19)/2 or 15x6/2+15x11+15x2/2 x = 225 m	M1 M1 A1 [3]	Accounts for totality of car journey (may be implied) Idea area = distance **SR** Accept 15x(13+17)/2 M1M1
	iii		Walks = 20x(-)2 = (-)40 m Jogs = 40/5 = 8 s T_s = 60 – ({6+11+2} + 20 + 8) T_s = 13 s	M1 A1 M1 A1 [4]	Finds distance walked T_s + ({6+11+2} + 20 + 8) = 60, needs all time elements

5	**i**		$V_P = 3 – 2.5x0.4$ (= 2) $V_Q = 2.5x0.4$ (= 1) +/-(0.5x2 - 0.2x1) (=+/-0.8) 0.5x2 - 0.2x1 = 0.5v + 0.2x3.2 (v = 0.32) 0.32 ms^{-1} up	M1 A1 B1 M1 A1 [5]	Calculation of either speed, either directions, |a|=2.5 Both magnitudes correct (disregard signs) Momentum before Uses conservation of momentum in collision (not both $v_P = 3$ and $v_Q = 0$) Accept "same", value positive
	ii		$V_Q = 3.2 – 2.5x0.6$ (=1.7) $V_R = 2.5x(0.4+0.6)$ (= 2.5) 0.2x1.7 – 0.3x2.5 = (0.2+0.3)v (v = -0.82) 0.82 ms^{-1} down	M1 A1 M1 A1ft A1 [5]	Calculation of either speed with its correct time, |a|=2.5 Both magnitudes correct (disregard signs) Uses momentum conservation in collision (not both $v_Q = 3.2$ and $v_R = 0$) LHS different signs, RHS same signs, ft cv(speeds Q, R) Value positive
6	**i**		"...smooth ring...", "..no friction at ring.."	B1 [1]	If a variety of reasons is offered, "smooth ring" must be the last
	ii		Tcosθ +5 = Tcos(90-θ) Tcosθ +5 = Tsinθ …………..(a) Tsinθ + Tsin(90-θ) = 7 Tsinθ + Tcosθ = 7 ………… (b)	M1 A1 M1 A1 [4]	"Resolves horiz" equation, needs TCorSθ, 3 terms, 2 of which are T resolved "Resolves vert" equation, needs TCorSθ, 3 terms, 2 of which are T resolved {Allow candidates solving for (iii) to begin in (ii)}
	iii		uses (b)+(a) and (b)-(a) for example Tsinθ = 6 or 2Tsinθ = 12, Tcosθ = 1 or 2Tcosθ = 2 $T^2 = 6^2 + 1^{(2)}$ T = 6.08 N Tanθ = 6(/1) θ = 80.5° *OR* (b) gives T=7/(sinθ+cosθ), subs in (a) for example 12cosθ = 2sinθ then mark as 6(iii) below for D*M1 A1 D*M1 A1	M1* A1 D*M1 A1 D*M1 A1 [6] M1* A1	Attempts to solve 2 equations in 2 unknowns Both terms have values correct Accept √37, 6.1 Uses a correct trig identity Accept 81°, 1.4 rad, 1.41 rad Attempts to solve 2 equations in 2 unknowns Correct two term equation in one variable

7	**i**		v = dx/dt v = $0.3t^2$ - 0.6t + 0.2 a = dv/dt a = 0.6t – 0.6	M1 A1 M1 A1ft [4]	Uses differentiation of x Uses differentiation of v Correct differentiation of candidate's v(t)
	ii		0.6t - 0.6 = 0 (t = 1) x(1) = $0.1\times1^3 - 0.3\times1^2 + 0.2\times1$ x(1) = 0 AG *OR* $0.1t^3 – 0.3t^2 + 0.2t = 0$ (t=1, and disregard others) a(1) = 0.6x1 -0.6 a(1) = 0	M1* D*M1 A1 [3]	Attempts to solve a=0 Puts solution in x formula Attempts to solve x=0 Puts solution in a formula
	iii		$0.3t^2$ - 0.6t + 0.2 = 0 t = 0.423 s t = 1.58 s	M1 A1 A1 [3]	Attempts to solve 3 term QE v = 0, accept imperfect attempt at formula, completing square or factorisation Accept 1 - 1/√3, 0.42, 0.422, or better Accept 1 + 1/√3, 1.6, 1.57, or better
	iv		x = $\int 0.2t^2 - 0.4 dt$ x = $0.2t^3/3 – 0.4t$ (+k) $0.1t^3 – 0.3t^2 + 0.2t = 0.2t^3/3 - 0.4t$ (+k) $t^3 - 9t^2 + 18t = 0$ $t^2 – 9t + 18 = 0$ AG (t-3)(t-6)=0 T = 3 s	M1* A1 D*M1 D*M1 A1 M1 A1 [7]	Uses integration, ignore omission of k x = $2t^3/30 – 4/10$ t (+k), or coeff t^3 0.067 or better Equates expressions for distance 3 terms with different powers of t, no constant Explains T is non-zero, or explains division by t Tries to solve given quadratic, accept imperfect attempt at completing square, formula or factorisation, and chooses smaller positive root
			Total	**[72]**	

Continued

Question 6 specifies the method students are likely to find most helpful. A more sophisticated approach, resolving parallel and perpendicular to the string, is mathematically valid, and leads to correct solutions. If seen it should be marked according to the following scheme, and no penalty levied.

The final 4 marks, in 6(iii), use the same mathematics as may be encountered when choosing an unorthodox method for solving the two simultaneous equations generated in 6(ii) of the specified method (see 6(iii) above).

			OR		
6	**i**		"...smooth ring...", "..no friction at ring.."	B1 [1]	If a variety of reasons is offered, "smooth ring" must be the last
	ii		T = 7cosθ + 5sinθ …….(a) T = 7sinθ – 5cosθ ……..(b)	M1 A1 M1 A1 [4]	Resolves //AR (Need not create Tcos/sinθ) Resolves //BR (Need not create Tcos/sinθ)
	iii		Equating expressions for T from (a) and (b) 2sinθ = 12cosθ tanθ = 6(/1) θ = 80.5° T = 7cos80.5 + 5sin80.5 or 7sin80.5 – 5cos80.5 T = 6.08	M1* A1 D*M1 A1 D*M1 A1 [6]	Attempts to solve 2 equations in 2 unknowns Correct two term equation in one variable Uses a correct trig identity Accept 81°, 1.4 rad, 1.41 rad Accept √37, 6.1